CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **October 2015**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) Sketch the graticule appearance of the Universal Transverse Mercator (UTM) projection for Zone 10 and label the following on the sketch: longitude of Central Meridian, Equator, False Northing and False Easting coordinates, latitude limits, longitudes of zone boundaries, scale factor at the Central Meridian.	12	
	b) What are the ellipsoidal (latitude and longitude) coordinates of the points where the meridian convergence values are minimal and maximal in UTM projections? Calculate the meridian convergence values corresponding to those points.	10	
	c) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM projections. What is that scale factor distortion?	6	
2.	Answer the following with regard to Transverse Mercator (TM) projection of an ellipsoid onto a plane.		
	a) Explain two specific conditions that the projection must satisfy along the Central Meridian with regard to the x and y projection coordinates.	3	
	b) Explain how the meridian convergence and scale factor variations in Transverse Mercator projections mathematically relate to those in the Universal Transverse Mercator projections.	2	
	c) Explain how the x and y coordinates in Transverse Mercator projections mathematically relate to those in the Universal Transverse Mercator projections.	3	
3.	A new survey point Q was established in the ITRF2008 (epoch 2012.16) using GNSS. After an appropriate transformation, the ITRF96 (epoch 2012.16) coordinates were determined for the point as $X = -4{,}793{,}404.167$ m; $Y = 407{,}107.994$ m; Z = $-4{,}175{,}081.559$ m. The velocity model for ITRF2008 (epoch 2012.16) are $v_x = -0.0196$ m/yr, $v_y = 0.0277$ m/yr, and $v_z = 0.0250$ m/yr; and other parameters are zero.		
	a) Calculate the ITRF96 (epoch 2000.0) coordinates of the survey point Q.	6	
	b) What are the significance of “2008” and “epoch 2012.16” in ITRF2008 (epoch 2012.16)?	4	
	c) Explain what constitutes an ITRF realization.	2	

4.	Answer the following:		
	a) Explain three important reasons why the GPS coordinate values, which are usually presented in the (X, Y, Z) geocentric coordinate system, are inconvenient for use by surveyors.	6	
	b) Clearly explain two important differences between CGVD28 and CGVD2013 (do not be tempted to state, for example, that one is … and the other is not).	6	
	c) According to Torge in his *Geodesy*, the Celestial Reference System (CRS) is an approximation to an inertial system. What is an inertial system? Explain the need for it in Geomatics.	4	
	d) Explain one important difference between the International Celestial Reference System (ICRS) and the International Terrestrial Reference System (ITRS) and name the parameters that are considered in transforming from one system to the other.	5	
	e) What is isometric latitude? Describe how it compares with the geodetic latitude from the Equator to the Pole and explain its important uses in map projection.	6	
	f) What is (T-t) correction? This correction is composed of two parts in conformal stereographic double projections; explain the two parts.	4	
5.	a) Explain two fundamental roles of time in positioning applications.	4	
	b) Define the Local Astronomic coordinate system with regard to its origin and the orientation of its axes.	2	
	c) Determine (with reasons for each case) which of the following would be affected if the (X, Y, Z) axes of the Conventional Terrestrial Reference system are rotated about the Z-axis: astronomic latitude, astronomic longitude, astronomic azimuth.	2	
	d) Explain what natural coordinates are and how they can be determined for a point.	4	
6.	In the *Conformal Map Projections in Geodesy* by Krakiwsky (1973), the modified direct mapping equations for Lambert conformal conic projection can be given as $x = r \sin \ell\lambda$ $y = r_0 - r \cos \ell\lambda$ where x and y are the map projection coordinates.		
	a) Clearly explain, with regard to the two equations, what r, r_0, ℓ and λ represent, including their possible uses. What types of territories are most suitable for conic projections?	7	
	b) Describe the magnitude of convergence of meridians in a conic projection.	2	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1 x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{\Delta\lambda^2}{2(206265)^2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value

Grid convergence, $\gamma_B = \Delta\lambda \sin\phi\left[1 + \frac{\Delta\lambda^2 \cos^2\phi}{3(206265)^2}\right]$; where $\Delta\lambda = (\lambda - \lambda_0)$ (in arc-seconds)

for any given longitude λ with central longitude at λ_0.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

Distortion Formulas:

Given: $X = f(\phi, \lambda)$ $\quad$ $Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \qquad m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi}; \qquad p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p'$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}} \quad ; \qquad \tan\mu_m = \frac{f_\varphi}{g_\varphi}$$

$$\tan\mu_s = \frac{g_\phi \cos\phi\cos A + g_\lambda \sin A}{f_\phi \cos\phi\cos A + f_\lambda \sin A} \quad ; \qquad \tan(180^\circ - A') = \frac{\tan\mu_m - \tan\mu_s}{1 + \tan\mu_m \tan\mu_s}$$

ITRF:

$$\mathbf{r}(\mathrm{t}) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(\mathrm{t} - \mathrm{t}_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.