CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **October 2015**

Note: This examination consists of 7 questions and formulae on 8 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Q. No	Time: 3 hours	Marks Value	Marks Earned
1.	Often, provincial or other authorities require that measurements, s_{ij}, by an EODMI or total station be done on a calibration baseline that has known pillar coordinates, $\underline{\mathbf{x}} = [x_1, x_2, \ldots, x_n]^T$. Most baselines were established several decades ago when EODMI were first being used and microwave instruments were more common. Rather than using a calibration baseline, some aspects of EODMI behaviour can be investigated by using a collinear array of points such as a series of tribrachs on tripods. ISO standard 17123-4 requires an array of 7 points [21 one-way distances being observable] with spacing following the Heerbrugg design, as explained by Rüeger in his *Introduction to Electronic Distance Measurement*. The spacing is based on the unit length [$U = \lambda_{mod}/2$] of the EODMI and on the overall length of the array, which is usually at least as long as any intended use of the EODMI. Usually with a collinear array, the $\underline{\mathbf{x}}$ would not be known *a priori*. In either the baseline or the array, the redundancy allows for a least squares linear parametric estimation. Normally, any corrections within the instrument [e.g. prism constants and meteorological "ppm"] are set to zero. Compare the use of a 7 point collinear array to the use of a 7 pillar calibration baseline under each of the following considerations:		
	a) How the layout of a linear array for a particular EODMI would differ from an established baseline;	2	
	b) What corrections are applied as "pre-processing" [i.e., before the estimation] to the values that are raw output from the EODMI and why;	2	
	c) What quantities are estimated and a typical observation equation [relating observables to estimables] with an explanation of the variables;	4	
	d) Explain the statistical tests that are usually done *a posteriori* [null and alternate hypotheses, statistic, test].	2	
	e) Using a 7 point linear array, a value of -0.93 mm was estimated for the additive constant. Its standard deviation was estimated to be ±0.43 mm. Is the additive constant significant at 95%?	2	
	f) Explain how the results of the estimation and the statistical tests would be used in subsequent measurements by the EODMI;	1	
	g) Explain the advantages and disadvantages of using a 7 point collinear array rather than a 7 point calibration baseline.	2	

2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 61°37'N. At a depth of 1.0 km, a horizontal adit runs easterly approximately in a straight line. A flat traverse follows the adit with stations along one side and included angles nearly 180°. Gyro-azimuths, using the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 3''$. Explain the corrections, with suggestions of their values [especially signs], which should be applied to an A_g, observed at 4.9 km [easterly] from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	10	
3.	The geometric behaviour of a sensitive structure can be monitored by the repeated measurements [directions, distances, zenith angles, height differences] of a local geodetic network involving object points [possibly occupied but at least sighted] and reference stations [occupied]. The object points are on the sensitive structure and are expected to be moving in some pattern. The reference stations are located supposedly beyond the influence of the forces on the structure and are expected to be stable [i.e., not moving]. The first network measurement campaign and coordinate estimation provides the basis for describing the movement of the structure over the period of time to the next, or later, subsequent campaigns. A description of the deformation comprises the vectors of displacement for the object points. For any one point, at time t_j: $\mathbf{d} = [d_x, d_y, d_z]^T$ with $d_x = x_j - x_1$, $d_y = y_j - y_1$, $d_z = z_j - z_1$, and $t_j > t_1$. By considering network datum defects and configuration defects as well as the expectation of stable reference points, explain the concerns that arise when following this approach to describing deformation and what should be done about these concerns.	10	
4.	The driving of a nearly horizontal adit [3.0 m diameter] from a shaft [3.0 m diameter] at a depth of 500 m is to be controlled using a series of instrument supports which will be along one side of the adit, 0.25 m out from its wall, and spaced every 200 m. The drive is to extend 1000 m from the shaft. Explain how you would transfer orientation and horizontal position with respect to a surface grid, down the shaft to the adit, and then along the adit to control the driving. Your explanation should have regard for equipment and technique, random errors and systematic influences, quality assurance measures, and should provide an "estimate" of the uncertainty associated with the horizontal positioning of the end point [1000 m from the shaft] in the surface grid system.	10	
5.	On the shelf in the company's survey stores, you have found a total station that has not been used for at least 10 years. The manufacturer's claim, following DIN 18723 [or ISO 17123, now], is an angular "accuracy", horizontally or vertically, of ± 1″ and a distance "accuracy" of ± 1 mm ± 1 ppm. Since there is no record of any testing or calibration of this particular instrument, explain the steps that you would recommend following to determine whether this total station is capable of behaving as the manufacturer claimed or better.	15	

6.	As part of a special traverse of "n" angles around a city block, a total station is to be set up along one side of the block, at one station with sight distances of 35 m and 255 m with the angle being very close to 180°. The 35 m sight is nearly horizontal but the 255 m sight is at a slope of 25%. These are the extreme values for this situation. The block angular misclosure is not to exceed $n^{1/2}10''$. "Not to exceed" is to be regarded as being at 99%. The values taken in the calculation of the misclosure would be the averages from at least two sets [a set being the average from face left and face right sightings]. a) Accounting for the effects of centering, leveling, pointing, and reading at this occupation, recommend an instrument that would be capable of meeting the misclosure requirement. Your design should consider observing in two or more sets. b) Explain which of centering, leveling, pointing, or reading is the dominant influence in this occupation and why.	15 5	
7.	Surveys and Mapping Branch [1978], in its *Specifications and Recommendations for Control Surveys and Survey Markers*, specifies that Special Order Levelling should use an optical mechanical instrument [sensitivity of $10''/2mm$, magnification of at least 40x] with a parallel plate micrometer along with double scaled invar staves [usually with 10mm graduations]. Lengths of sight are not to exceed 50m and are to be balanced within 5m. Also, for Special Order, the discrepancy between independent forward and backward levellings is not to exceed [i.e., $1-\alpha = 99\%$] $\pm 3mmK^{1/2}$ with K in kilometres. First Order levelling is similar but with 32x, 60m, 10m, and $\pm 4mmK^{1/2}$. Second order is $\pm 8mmK^{1/2}$ following at least First Order, possibly with $20''/2mm$. A contract may require determining local benchmark elevations following Special, First, or Second Order, depending on the use of the benchmarks. A height difference is the average of the forward and backward levellings. a) Derive an expression, based on the discrepancy and as a function of K, for the standard deviation of a height difference and calculate the expected standard deviation for each of the three orders. b) Using modern total stations [angular "accuracy", horizontally or vertically, of $\pm 1''$ and a distance "accuracy" of ±1mm ±1ppm] has the potential for competing with traditional differential levelling, especially when considering the limitation on lengths of sight consequent of level lines of sight. Explain whether this is possible and at what order [Special, First, or Second].	5 15	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.90	0.95	0.975	0.99	0.995
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84
13	19.81	22.36	24.74	27.69	29.82
14	21.06	23.68	26.12	29.14	31.32
15	22.31	25.00	27.49	30.58	32.80

Percentiles of the t distribution:

	0.90	0.95	0.975	0.99	0.995
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831

22 1.321 1.717 2.074 2.508 2.819

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h; \sqrt{\sigma_c^2} = \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2div; \ \sigma_l = \pm 0.02div; \ \sigma_l \le \pm 0.5''$$

$$\sigma_{\beta_l} = \pm \sigma_l \sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm \frac{30''}{M} \le \sigma_p \le \pm \frac{60''}{M}; \qquad \sigma_{ps} \approx \frac{70''}{M}$$

$$b = 2a + c; \quad a = \frac{120}{206264.8}\frac{D}{M}; \quad 2'' \le c \le 4''$$

$$\sigma_r \ge \pm 0.3div; \sigma_r = \pm 0.3div; \sigma_r = \pm 2.5div; \sigma_r = \pm 0.6''$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm \sigma_l$$

$$\sigma_{z_p} = \pm \frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm \frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1 \sin\alpha_1}{a}; \quad \sin\beta_2 = \frac{b_2 \sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\sigma_{\beta_i}^2; \quad \sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_{i+1} - x_i)^2\sigma_{\alpha_i}^2$$

$$\sigma_{x_n}^2 = \sigma_s^2 \sum_{i=1}^{n}\left(\frac{x_i - x_{i-1}}{s_i}\right)^2$$

$$\sigma_{x_n}^2 = \sum_{i=1}^{n-1}(y_n - y_i)^2\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{x_{i+1} - x_i}{s_i}\right)^2\sigma_{s_i}^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1} (x_n - x_i)^2 \sigma_{\beta_i}^2 + \sum_{i=1}^{n-1} \left(\frac{y_{i+1} - y_i}{s_i} \right)^2 \sigma_{s_i}^2$$

$$\sigma_{x_n y_n} = \sum_{i=1}^{n-1} (y_n - y_i)(x_n - x_i) \sigma_{\beta_i}^2 + \sum_{i=1}^{n-1} \left(\frac{(x_{i+1} - x_i)(y_{i+1} - y_i)}{s_i^2} \right) \sigma_{s_i}^2$$

$\sigma_s^2 = a^2 + b^2 s^2$

$$d\delta = 8'' \frac{pS}{T^2} \frac{dT}{dx}$$

1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K

$$T = \frac{\sum_{i=1}^{n} \left[(h_{i+1} - h_i)(T_i + T_{i+1}) \right]}{2(h_n - h_1)}$$

$$\Delta h_w = \frac{w}{aE} \left(Lh - \frac{h^2}{2} \right)$$

$$n_a = 1 + \frac{0.359474(0.0002945)p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474(0.0002821)p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\varepsilon_A = \frac{206264.8}{b} \sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2} \right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE}; \qquad E = 2.1 \times 10^6 \text{ kgcm}^{-2}$$

$$T = 2\pi \sqrt{\frac{H}{g}}; \qquad g = 980 \text{ cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1(r_1 - r_2)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

N = N' + ΔN; ΔN = caΔt; E = A - A_g = t ± γ - A_g

$$\alpha_1 = \mathrm{A} - \eta \tan\phi$$

$$z = Z + [\xi \cos\alpha_1 + \eta \sin\alpha_1]$$

$$\alpha_2 = \alpha_1 + [\eta \cos\alpha_1 - \xi \sin\alpha_1]\cot z$$

$$\alpha_3 = \alpha_2 + \frac{h}{M_m} e^2 \sin\alpha_2 \cos\alpha_2 \cos^2\phi_{TO}$$

$$\alpha = \alpha_3 - \frac{e^2 s^2 \cos^2\phi_m \sin 2\alpha_3}{12N_m^2}$$

$$t = \alpha - \gamma - [T - t]$$

$$t = \alpha - \theta - [T - t]$$

$$\theta = \frac{d \tan\phi (1 - \varepsilon^2 \sin^2\phi)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E \tan\phi}{R}$$

6378206.4 m, 0.0822718948; 6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell} = \frac{\Delta s}{s}$$

$$d_x = r_{x_1} - r_{x_2}; \quad d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \quad \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i; \; c_{align} = -\frac{d^2}{2s}; \; c_{temp} = \alpha(t - t_0)s; \; c_{tens} = \frac{P - P_0}{aE} s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right); \quad c_{sea} = \frac{H}{R + H} s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha} ?; \quad \frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}} ?; \quad \left|\frac{a_\mu}{s_{a_\mu}}\right| \le t_{\nu,1-\frac{\alpha}{2}} ?$$

$$\left|\frac{\hat{r}_i}{\sigma_{\hat{r}_i}}\right| \le n(0,1), 1-\frac{\alpha}{2}; \qquad \left|\frac{\hat{r}_i}{\hat{\sigma}_{\hat{r}_i}}\right| \le \tau, \nu, 1-\frac{\alpha}{2}, \quad \tau_\nu = \frac{\sqrt{\nu}}{\sqrt{\nu-1+t^2_{\nu-1}}} t_{\nu-1}$$

$$C_x = \sigma_0^2\left[C_{x_S}^{-1} + \left(A^T PA\right)_U\right]^{-1}$$

$$\Delta_{f/b} \le \pm 3mm\sqrt{K}; \ \Delta_{f/b} \le \pm 4mm\sqrt{K}; \ \Delta_{f/b} \le \pm 8mm\sqrt{K}; \ \Delta_{f/b} \le \pm 24mm\sqrt{K}$$

$$\sigma_{r_l} = \pm d\sigma_l; \qquad \sigma_{r_{pr}} = \pm\frac{45''}{M}d, \quad d > 20m; \quad \sigma_{r_{pr}} = \pm\frac{30''}{M}d, \quad d \le 20m$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$

$$s_{ij} + z_0 = x_j - x_i; ks_{ij} + z_0 = x_j - x_i; s = s' + s'\Delta N$$

$$n_{obs} = \frac{n_{pts}\left(n_{pts}-1\right)}{2}$$

`c+r = 0.0675 K`^{2}

$$A = iU; \quad B_0 = \frac{1}{15}[C_0 - 6A - U]; \quad D = \frac{U}{36}$$

$1to2 : A + 1B + 3D$

$2to3 : A + 3B + 7D$

$3to4 : A + 5B + 11D$

$4to5 : A + 4B + 9D$

$5to6 : A + 2B + 5D$

$6to7 : A + D$

$$d_4 = 2R\arcsin\sqrt{\frac{R^2\sin^2(d_1\frac{k}{2R}) - k^2\frac{(H_2-H_1)^2}{4}}{k^2(R+H_1)(R+H_2)}}$$

$$d_4 = R\arctan\left[\frac{d_2\sin(z_1+\varepsilon_1+\delta)}{R+H_1+d_2\cos(z_1+\varepsilon_1+\delta)}\right]$$

$$\hat{y} = a + bx; \qquad x = \frac{-a}{b} + \frac{\hat{y}}{b}; \quad x = z_0 + k\hat{y}$$

$$s_a = s_0 \sqrt{\frac{\sum x^2}{n\sum x^2 - \left(\sum x\right)^2}} \quad and \quad s_b = \frac{s_0}{\sqrt{\frac{\sum x^2 - \left(\sum x\right)^2}{n}}} \quad with$$

$$\hat{\sigma}_0^2 = s_0^2 = \frac{\sum (a + bx - y)^2}{n-2}$$

$$\hat{\sigma}_{z_0}^2 = \hat{\sigma}_0^2 \frac{6}{(N-1)(N-2)} \quad with \quad \hat{\sigma}_0^2 = \frac{\sum v^2}{n-u}$$