CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS October 2015

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 9 questions on 3 pages. **Marks**

<table>
<tr><th>Q. No</th><th>Time: 3 hours</th><th>Value</th><th>Earned</th></tr>
<tr><td>1.</td><td>Explain briefly the difference between:
a) Precision and accuracy;
b) Type I and Type II errors in statistical testing;
c) Statistically independent and uncorrelated;
d) Standard deviation and root mean square error.</td><td>10</td><td></td></tr>
<tr><td>2.</td><td>Given the following mathematical model

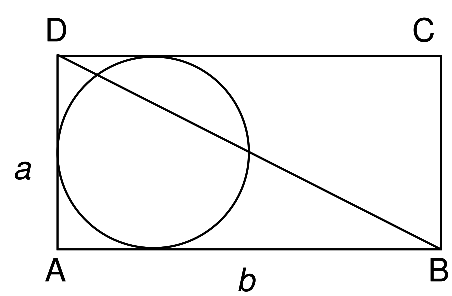
$f(\ell, x) = 0 \quad C_\ell \quad C_x$

where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix.

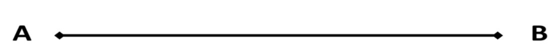
a) Derive the least squares normal equation.
b) Derive the least squares solution of the unknown parameters and their variance-covariance matrix.</td><td>15</td><td></td></tr>
<tr><td>3.</td><td>Sides a and b are measured once each with different precisions as follows:

$\boldsymbol{l} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \text{m}$

$\boldsymbol{C}_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \text{cm}^2$



a) Estimate the areas of triangle ABD and the circle shown inside the rectangle.
b) Estimate the standard deviations of the quantities computed in Part a).
c) Estimate the correlation between the triangle and the circle estimates.
d) Discuss the nature of the correlations computed in Part c).</td><td>15</td><td></td></tr>
<tr><td>4.</td><td>The distance between Point A and Point B has been independently measured 5 times with the same precision using a distance measuring device and the standard deviation of the obtained mean distance is 1.58cm. Determine the precision of the distance measurement.


</td><td>5</td><td></td></tr>
</table>

5. Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

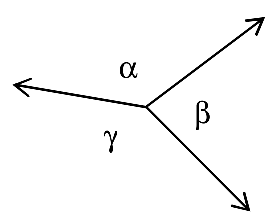
$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \; m^2$$

10

6. Given the angle measurements at a station along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:

a) the estimated residuals;
b) the variance-covariance matrix of the estimated residuals;
c) the estimated observations;
d) the variance-covariance matrix of the estimated observations;
e) the estimated variance factor.

Angle	Measurement	Standard Deviation
α	134°38'56"	6.7"
β	83°17'35"	9.9"
γ	142°03'14"	4.3"



15

7. Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities:

a) the estimated parameters;
b) the variance-covariance matrix of the estimated parameters;
c) the estimated difference between α and β;
d) the variance of the estimated difference between α and β.

10

8. Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,cm^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,cm$, conduct a statistical test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

10

9.	A baseline of calibrated length (μ) 200.00m is measured 5 times independently with the same precision. The obtained distance measurements are given in the following table: d_1 = 200.02m, d_2 = 199.93m, d_3 = 199.98m, d_4 = 199.99m, d_5 = 200.01 Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table:	10	
	Total Marks:	100	

d_1	d_2	d_3	d_4	d_5
200.02m	199.93m	199.98m	199.99m	200.01

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36