CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **October 2014**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	A new survey point Q was established in the ITRF2008 (epoch 2012.16) using GNSS. After an appropriate transformation, the ITRF96 (epoch 2012.16) coordinates were determined for the point as *X = -4,793,404.167 m*; *Y = 407,107.994 m*; Z = *-4,175,081.559 m*. The velocity model for ITRF2008 (epoch 2012.16) are v_x = *-0.0196 m/yr*, v_y = *0.0277 m/yr*, and v_z = *0.0250 m/yr*; and other parameters are zero.		
	a) Calculate the ITRF96 (epoch 2000.0) coordinates of the survey point Q.	6	
	b) What are the significance of "2008" and "epoch 2012.16" in ITRF2008 (epoch 2012.16)?	4	
	c) Explain what constitutes an ITRF realization.	2	
	d) The North American Datum of 1983 (NAD83) is currently aligned with a fixed epoch of an ITRF realization to support most spatial users. Clearly explain one important practical limitation with aligning NAD83 with a fixed epoch.	3	
2.	The map coordinates of points R and S in a stereographic double projection and the UTM projection are as follows:		

	Stereographic double projection	**UTM projection**
Point R:	Northing = 7,528,656.555 m Easting = 2,497,692.180 m Latitude: 46° 45′ 28.0998″ N Longitude: 66° 31′ 48.7559″ W Grid Convergence = -0° 01′ 19.06″ (T-t) Correction = 0.04″	Northing = 5,181,212.674 m Easting = 688,610.493 m Latitude: 46° 45′ 28.0998″ N Longitude: 66° 31′ 48.7559″ W
Point S:	Northing = 7,519,393.255 m Easting = 2,498,724.831 m	Northing = 5,171,985.701 m Easting = 689,937.055 m

Q. No		Value	Earned
2.	a) Calculate the plane bearings of line RS in the stereographic double projection and UTM projection. Explain two important reasons why the two calculated plane bearings are different.	4	
	b) Calculate (independently) the geodetic bearings of line RS from the plane bearings computed for the stereographic double projection and the UTM projection (assuming the mean radius of the earth in the mapping region, R_m = 6,382,129.599 m). Explain one important reason why the two calculated geodetic bearings are different.	18	

3.	According to Torge in his *Geodesy*, the Celestial Reference System (CRS) is an approximation to an inertial system. a) What is an inertial system? Explain the need for it in Geomatics. b) Describe the three-dimensional Cartesian coordinate system and the spherical coordinate system, for CRS.	 4 6	
4.	Answer the following: a) Explain three important reasons why the GPS coordinate values, which are usually presented in the (X, Y, Z) geocentric coordinate system, are inconvenient for use by surveyors. b) What is Tissot indicatrix? Describe the conformal and the equal-area map projections with regard to the possible sizes, shapes and orientations of Tissot indicatrices on them. c) What is the purpose of seven parameter transformation? Name the parameters involved in the seven parameter transformation. d) Clearly explain one important difference between CGVD28 and CGVD2013 (do not be tempted to state, for example, that one is … and the other is not).	 6 9 5 3	
5.	a) Explain two fundamental roles of time in positioning applications. b) Explain the differences between the Local Astronomic coordinate system and the Horizon coordinate system with regard to their origins, fundamental planes, and how an object can be positioned in each. c) Explain what natural coordinates are and how they can be determined for a point.	4 6 4	
6.	Using well labeled sketches only, illustrate the Mercator and the Polar Stereographic projections in the Northern hemisphere; give one sketch for the Mercator projection and the other sketch for the Polar Stereographic projection. The sketches must show the representations (in dotted lines) of **loxodrome with bearing 90°**, and the projections (in bold lines) of the **Equator**, **Central Meridian**, **parallels** and **meridians** with the appropriate relationship between the lines of the graticule clearly illustrated.	16	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - 500{,}000$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = 0.9996\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - 500{,}000$; $x_u^2 = x_1^2 + x_1 x_2 + x_2^2$

Grid convergence, $\gamma_B = \Delta\lambda \sin\phi \left[1 + \frac{\Delta\lambda^2 \cos^2\phi}{3(20265)^2}\right]$; where $\Delta\lambda = (\lambda - \lambda_0)$ (in arc-seconds) for any given longitude λ with central longitude at λ_0.

$$\alpha = t + \gamma + (T - t)$$

$$X_f = X_G \times K_f + X_0; \qquad Y_f = Y_G \times K_f$$

$$Sf = \frac{R_m}{R_m + H_m}$$

$$x = (N + h)\cos\phi\cos\lambda; \qquad y = (N + h)\cos\phi\sin\lambda$$

$$z = \left[(1 - e^2)N + h\right]\sin\phi; \qquad N = \frac{a}{\sqrt{(1 - e^2\sin^2\phi)}}$$

$$\Delta r^{LG} = R_3(\eta_0 \tan\phi_0)R_2(-\xi_0)R_1(\eta_0) \times \Delta r^{LA}$$

$$\Delta r^{G} = R_3(\pi - \lambda_0)R_2(\frac{\pi}{2} - \phi_0)P_2 \times \Delta r^{LG}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B^G = r^G + \Delta r^G$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + (1 + \delta k)\begin{bmatrix} 1 & \gamma & -\beta \\ -\gamma & 1 & \alpha \\ \beta & -\alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\alpha_{ij} = A_{ij} - \eta_i \tan\phi_i - (\xi_i \sin\alpha_{ij} - \eta_i \cos\alpha_{ij})\cot Z_{ij}$$

<u>Distortion Formulas:</u>

Given: $X = f(\phi, \lambda) \qquad Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \qquad m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi}; \qquad p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A'_p$$

$$\sin A'_p = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}} \quad ; \qquad \tan\mu_m = \frac{f_\varphi}{g_\varphi}$$

$$\tan\mu_s = \frac{g_\phi \cos\phi\cos A + g_\lambda \sin A}{f_\phi \cos\phi\cos A + f_\lambda \sin A} \quad ; \qquad \tan(180^\circ - A') = \frac{\tan\mu_m - \tan\mu_s}{1 + \tan\mu_m \tan\mu_s}$$

<u>ITRF:</u>

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.