CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **October 2014**

Note: This examination consists of 7 questions and formulae on 8 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	The survey markers [2.5 cm square by 1.25 m long] at the corners of a 250 m by 250 m square parcel were coordinated by the combination of a three dimensional total station traverse from corner to corner along with the static GPS occupation of two diagonally opposite corners connecting them, at 2 cm at 95%, to provincial coordinated monuments [Easting and Northing, elevation only ±1 m]. Because of obstructions, each side connection by total station required two intermediate setups equally spaced along a traverse route of 270 m. The linear misclosure of the traverse did not exceed 1 in 5000. A structure is to be located within a square, 50 m by 50 m, central to the parcel. The setting out of prefabricated structure components requires three dimensional coordinating within the square with relative error of better than 5 mm. The statements “did not exceed” and ‘better than” should be interpreted at 99%. Obstacles within the parcel restrict the lines of sight to be clear only to the two nearest corners of the parcel [i.e., those at the ends of the closest side]. Explain whether the existing coordination of the survey markers would be suitable as control. If so, also explain how they should be used. If not, also explain what additional measurement efforts, and possible monumentation, would be necessary and then how the product of those efforts should be used. Explaining how they should be used should include a discussion of the geometry of any connections with a sketch and of the equipment and technique with associated precision. In either case, explain what checks [with statistical guidance] on subsequent measurements should be made to ensure the eventual relative error of better than 5 mm.	20	
2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 61°01'N. At a depth of 2.1 km, an adit runs approximately in an easterly direction. A flat traverse follows the adit with stations along one side. Gyro-azimuths, following the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 3''$. Explain the corrections, with suggestions of their values [especially signs], which should be applied to an A_g, observed at 3.5 km [easterly] from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	10	

3.	A campaign of observations, at t_1, can be adjusted to estimate the coordinates of the points involved, based on $\mathbf{l}_1 + \mathbf{v}_1 = \mathbf{A}_1\mathbf{x}_1$ ["**l**" etc. (bold lower case) denote vectors, "**A**" (bold) denotes a matrix]. During a later campaign, at t_2, the observations can be repeated so that $\mathbf{l}_2 + \mathbf{v}_2 = \mathbf{A}_2\mathbf{x}_2$. If there are object points on a sensitive structure, its behaviour can be described geometrically with respect to the reference points, using the displacement field resulting from $\mathbf{d}_x = \mathbf{x}_2 - \mathbf{x}_1$, in which some of the elements of the $\mathbf{d}_x$ vector are for the object points, say $\mathbf{d}_{x\,obj}$. Under certain circumstances, it may be possible to difference the observations, $\mathbf{d}_l = \mathbf{l}_2 - \mathbf{l}_1$, so that the displacement field can be estimated, based on $\mathbf{d}_l + \mathbf{v}_d = \mathbf{A}\mathbf{d}_x$. a) Explain the conditions under which the $\mathbf{d}_x$ can be calculated in the coordinate differencing approach, with respect to the $\mathbf{l}_i$, $\mathbf{A}_i$ and $\mathbf{x}_i$, and the advantages and disadvantages of this approach. b) Explain the conditions under which the observation differencing approach may be followed and its advantages and disadvantages. c) Explain which approach can accommodate geotechnical data and give an example of an appropriate geotechnical observable, r_j, with its observation equation [relating observable to estimables] and an explanation of how the value recorded in the field, r'_i at t_i, becomes the "observation" value, r_j. d) If the monitoring were to endure over a long period of time, say several decades, explain what concerns would arise in each of the two approaches and how best to deal with those concerns.	3 3 6 3	
4.	A flat hanging traverse is to be measured with uniform sight lengths of 100 m ±2 mm. There are two "fixed" stations, "A" and "B", plus seven traverse stations, "P1" to "P7" so that "B" and "P1" to "P6" would be occupied while "A" and "P7" would be sighted. One approach is to measure the included horizontal angles [values near 180°] with $\sigma_\beta = \pm 5''$. An alternative method is to occupy certain stations and to observe the azimuth to the next station using a gyro attachment so that $\sigma_A = \pm 15''$. a) If only included angles were observed, explain the dominant component of the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty. b) If azimuths rather than included angles were observed, explain the dominant component of the random positional uncertainty at the end point of the traverse, "P7", and suggest a value and orientation of the uncertainty. c) If the traverse were along a tunnel, close to one wall, explain the dominant systematic influence and whether included angles or azimuths should be observed.	5 3 2	
5.	As part of a special traverse of "n" angles around a city block, a total station is to be set up along one side of the block, at one station with sight distances of 50 m and 250 m with the angle being very close to 180°. The 50 m sight is nearly horizontal but the 250 m sight is at a slope of 15%. These are the extreme values for this situation. a) Accounting for the effects of centering, leveling, pointing, and reading at this occupation, recommend an instrument that would be capable of meeting the requirement that the block angular misclosure is not to exceed $n^{1/2}10''$. "Not to exceed" is to be regarded as being at 99%. The values taken in the calculation of the misclosure would the averages from at least two sets [a set being the average of face left and face right sightings]. b) Explain which [centering, leveling, pointing, or reading] is the dominant influence in this occupation and why.	10 5	

6.	On the shelf in the company's survey stores, you have found a total station that has not been used for at least 20 years. The manufacturer's claim, following DIN 18723 [or ISO 17123, now], is an angular "accuracy", horizontally or vertically, of $\pm 2''$ and a distance "accuracy" of $\pm$ 2 mm $\pm$ 2 ppm. Since there is no record of any testing or calibration of this particular instrument, explain the steps that you would recommend following to determine whether this total station is capable of behaving as the manufacturer claimed.	15	
7.	Surveys and Mapping Branch [1978], in its _Specifications and Recommendations for Control Surveys and Survey Markers_, specifies that Special Order Levelling should use an optical mechanical instrument [sensitivity of 10″/2mm, magnification of at least 40x] with a parallel plate micrometer along with double scaled invar staves [usually with 10 mm graduations]. Lengths of sight are not to exceed 50 m and are to be balanced within 5 m. Also, for Special Order, the discrepancy between independent forward and backward levellings is not to exceed [i.e., $1-\alpha = 99\%$] $\pm 3\text{mm}K^{1/2}$ with K in kilometres. First Order levelling is similar but with 32x, 60 m, 10 m, and $\pm 4\text{mm}K^{1/2}$. Several instruments with appropriate staves are available: i. Wild N3 tilting level [42x, 10″/div (setting accuracy ±0.25″), σ_{ISO}: ±0.2 mm]; ii. Kern GK23 tilting level with micrometer [30x, 20″/div, σ_{ISO}: ±0.5 mm]; iii. Kern GK2A automatic level with micrometer [33x, σ_{ISO}: ±0.3 mm]; iv. Leica NA2 automatic level with micrometer [32x, σ_{ISO}: ±0.3 mm]; and v. Leica DNA03 digital automatic level [24x, setting accuracy ±0.3″, σ_{ISO}: ±0.3 mm] with bar coded invar staves. The "σ_{ISO}" is the standard deviation per 1 km of double run levelling according to ISO 17123-2. a) Consider each type of level and, with some numerical substantiation, explain whether it is suitable for Special Order levelling and, if not, whether for First Order. b) Explain how you would ensure that the lengths of sight are balanced in Special Order levelling [50 m ± 5 m] along a route with many successive setups.	10 5	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.90	0.95	0.975	0.99	0.995
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84

Percentiles of the t distribution:

	0.90	0.95	0.975	0.99	0.995
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h; \sqrt{\sigma_c^2} = \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2 div;\ \ \sigma_l = \pm 0.02 div;\ \ \sigma_l \le \pm 0.5"$$

$$\sigma_{\beta_l} = \pm\sigma_l\sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm\frac{30"}{M} \le \sigma_p \le \pm\frac{60"}{M};\qquad \sigma_{ps} \approx \frac{70"}{M}$$

$$b = 2a + c;\quad a = \frac{120}{206264.8}\frac{D}{M};\quad 2" \le c \le 4"$$

$$\sigma_r \ge \pm 0.3 div; \sigma_r = \pm 0.3 div; \sigma_r = \pm 2.5 div; \sigma_r = \pm 0.6"$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm\sigma_l$$

$$\sigma_{z_p} = \pm\frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm\frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1\sin\alpha_1}{a};\quad \sin\beta_2 = \frac{b_2\sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\sigma_{\beta_i}^2;\quad \sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_{i+1} - x_i)^2\sigma_{\alpha_i}^2$$

$$\sigma_{x_n}^2 = \sigma_s^2\sum_{i=1}^{n}\left(\frac{x_i - x_{i-1}}{s_i}\right)^2$$

$$\sigma_{x_n}^2 = \sum_{i=1}^{n-1}(y_n - y_i)^2\,\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{x_{i+1} - x_i}{s_i}\right)^2\sigma_{s_i}^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2\,\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{y_{i+1} - y_i}{s_i}\right)^2\sigma_{s_i}^2$$

$$\sigma_{x_n y_n} = \sum_{i=1}^{n-1}(y_n - y_i)(x_n - x_i)\sigma_{\beta_i}^2 + \sum_{i=1}^{n-1}\left(\frac{(x_{i+1} - x_i)(y_{i+1} - y_i)}{s_i^2}\right)\sigma_{s_i}^2$$

$\sigma_s^2 = a^2 + b^2s^2$

$$d\delta = 8'' \frac{pS}{T^2}\frac{dT}{dx}$$

1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K

$$T = \frac{\sum_{i=1}^{n}\left[(h_{i+1} - h_i)(T_i + T_{i+1})\right]}{2(h_n - h_1)}$$

$$\Delta h_w = \frac{w}{aE}\left(Lh - \frac{h^2}{2}\right)$$

$$n_a = 1 + \frac{0.359474(0.0002945)p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474(0.0002821)p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\varepsilon_A = \frac{206264.8}{b}\sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2}\right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE}; \qquad E = 2.1\text{x}10^6 \text{ kgcm}^{-2}$$

$$T = 2\pi\sqrt{\frac{H}{g}}; \qquad g = 980 \text{ cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1(r_1 - r_2)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

$N = N' + \Delta N$; $\Delta N = ca\Delta t$; $E = A - A_g = t \pm \gamma - A_g$

$$\alpha_1 = A - \eta \tan\phi$$

$$z = Z + [\xi \cos\alpha_1 + \eta \sin\alpha_1]$$

$$\alpha_2 = \alpha_1 + [\eta \cos\alpha_1 - \xi \sin\alpha_1]\cot z$$

$$\alpha_3 = \alpha_2 + \frac{h}{M_m} e^2 \sin\alpha_2 \cos\alpha_2 \cos^2\phi_{TO}$$

$$\alpha = \alpha_3 - \frac{e^2 s^2 \cos^2\phi_m \sin 2\alpha_3}{12 N_m^2}$$

$$t = \alpha - \gamma - [T - t]$$

$$t = \alpha - \theta - [T - t]$$

$$\theta = \frac{d \tan\phi \left(1 - \varepsilon^2 \sin^2\phi\right)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E \tan\phi}{R}$$

6378206.4 m, 0.0822718948;   6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell} = \frac{\Delta s}{s}$$

$$d_x = r_{x_1} - r_{x_2}; \quad d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \quad \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i; \; c_{align} = -\frac{d^2}{2s}; \; c_{temp} = \alpha(t - t_0)s; \; c_{tens} = \frac{P - P_0}{aE} s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right); \quad c_{sea} = \frac{H}{R + H} s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha}\,?; \quad \frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}\,?; \quad \frac{a_\mu}{s_{a_\mu}} \le t_{\nu,1-\frac{\alpha}{2}}\,?$$

$$\left|\frac{\hat{r}_i}{\sigma_{\hat{r}_i}}\right| \le n(0,1), 1-\frac{\alpha}{2}; \qquad \left|\frac{\hat{r}_i}{\hat{\sigma}_{\hat{r}_i}}\right| \le \tau, \nu, 1-\frac{\alpha}{2}, \quad \tau_\nu = \frac{\sqrt{\nu}}{\sqrt{\nu-1+t_{\nu-1}^2}} t_{\nu-1}$$

$$C_x = \sigma_0^2\left[C_{x_S}^{-1} + \left(A^T PA\right)_U\right]^{-1}$$

$$\Delta_{f/b} \le \pm 3mm\sqrt{K}; \;\; \Delta_{f/b} \le \pm 4mm\sqrt{K}; \;\; \Delta_{f/b} \le \pm 8mm\sqrt{K}; \;\; \Delta_{f/b} \le \pm 24mm\sqrt{K}$$

$$\sigma_{r_l} = \pm d\sigma_l; \qquad \sigma_{r_{pr}} = \pm\frac{45''}{M}d, \quad d > 20m; \quad \sigma_{r_{pr}} = \pm\frac{30''}{M}d, \quad d \le 20m$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$

$$s_{ij} + z_0 = x_j - x_i; \; ks_{ij} + z_0 = x_j - x_i; \; s = s' + s'\Delta N$$

$$n_{obs} = \frac{n_{pts}\left(n_{pts}-1\right)}{2}$$

$c+r = 0.0675\ K^2$

$$A = iU; \quad B_0 = \frac{1}{15}[C_0 - 6A - U]; \quad D = \frac{U}{36}$$

$1to2 : A + 1B + 3D$

$2to3 : A + 3B + 7D$

$3to4 : A + 5B + 11D$

$4to5 : A + 4B + 9D$

$5to6 : A + 2B + 5D$

$6to7 : A + D$

$$d_4 = 2R\arcsin\sqrt{\frac{R^2\sin^2(d_1\frac{k}{2R}) - k^2\frac{(H_2-H_1)^2}{4}}{k^2(R+H_1)(R+H_2)}}$$

$$d_4 = R\arctan\left[\frac{d_2\sin(z_1+\varepsilon_1+\delta)}{R+H_1+d_2\cos(z_1+\varepsilon_1+\delta)}\right]$$

$$\hat{y} = a + bx; \qquad x = \frac{-a}{b} + \frac{\hat{y}}{b}; \quad x = z_0 + k\hat{y}$$

$$s_a = s_0\sqrt{\frac{\sum x^2}{n\sum x^2 - \left(\sum x\right)^2}} \quad and \quad s_b = \frac{s_0}{\sqrt{\frac{\sum x^2 - \left(\sum x\right)^2}{n}}} \quad with$$

$$\hat{\sigma}_0^2 = s_0^2 = \frac{\sum(a+bx-y)^2}{n-2}$$

$$\hat{\sigma}_{z_0}^2 = \hat{\sigma}_0^2 \frac{6}{(N-1)(N-2)} \quad with \quad \hat{\sigma}_0^2 = \frac{\sum v^2}{n-u}$$