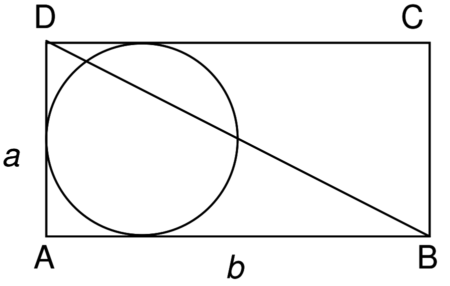
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS

October 2014

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 9 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Briefly explain the following terms: a) Precision b) Root mean square error c) Internal reliability d) Redundancy of a linear system e) Correlation coefficient	10	
2	Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^{n} \ell_i}{n}$. Each measurement ℓ_i is made with a standard deviation σ.	10	
3.	Sides *a* and *b* are measured once each with different precisions as follows: $\boldsymbol{l} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ m $\boldsymbol{C}_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ cm^2  a) Estimate the areas of triangle ABD and the circle shown inside the rectangle. b) Estimate the standard deviations of the quantities computed in Part a). c) Estimate the correlation between the triangle and the circle estimates. d) Discuss the nature of the correlations computed in Part c).	15	
4.	Given the following over-determined linear system: y = Ax $\quad C_y$ where y is the vector of observations and C_y is its variance-covariance matrix, x is the vector of unknown parameters, A is the design matrix. a) Derive the least squares normal equation. b) Derive the least squares solution of the unknown parameters and their variance-covariance matrix.	10	

5. Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

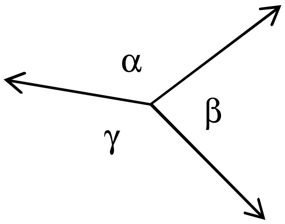
$$C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \ \text{m}^2$$

10

6. Given the angle measurements at a station along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:

a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor

Angle	**Measurement**	**Standard Deviation**
α	134°38’56”	6.7”
β	83°17’35”	9.9”
γ	142°03’14”	4.3”



15

7. Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities:

a) the estimated parameters
b) the variance-covariance matrix of the estimated parameters
c) the estimated difference between α and β
d) the variance of the estimated difference between α and β

10

8. Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,\text{cm}^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,\text{cm}$, conduct a statistical test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

10

9.

A baseline of calibrated length (μ) 200.00m is measured 5 times independently with the same precision. The obtained distance measurements are given in the following table.

d1	d2	d3	d4	d5
200.02m	199.93m	199.98m	199.99m	200.01

Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance.

The critical value that might be required in the testing is provided in the following table:

	t_{α}			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

10

Total Marks: 100