CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

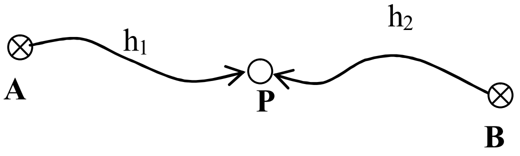
C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS

March 2016

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 3 pages. **Marks**

Q. No Time: 3 hours Value Earned

Q. No		Value	Earned
1.	Briefly explain the following terms: a) Precision b) Root mean square error c) Internal reliability d) Redundancy of a linear system e) Correlation coefficient	10	
2.	Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\ \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at P using least squares adjustment is equal to 2cm.  	10	
3.	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix a) Linearize the mathematical model b) Formulate the variation function c) Derive the least squares normal equation d) Derive the least squares solution of the unknown parameters.	15	

4. Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^{n} \ell_i}{n}$, each measurement ℓ_i is made with the same standard deviation σ.

10

5. Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

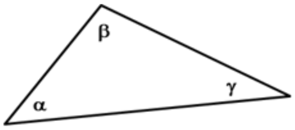
$$\mathrm{C_x} = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \ \mathrm{m}^2$$

10

6. Given the angle measurements at a station along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:

a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor

Angle	**Measurement**	**Standard Deviation**
α	104°38’56”	6.7”
β	33°17’35”	9.9”
γ	42°03’14”	4.3”



15

7. Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities:

a) the estimated parameters
b) the variance-covariance matrix of the estimated parameters
c) the estimated difference between α and β
d) the variance of the estimated difference between α and β

10

8. Given a minimum constraint leveling network with 100 observed height differences and 40 unknown points, use mathematical equations to explain which method of adjustment (parametric or conditional) you will recommend to be used for this problem.

5

9.

Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:

$$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and the function $x = \ell_1 + \ell_2$, determine C_x.

5

10.

An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value 45°00’00”.

α_1	α_2	α_3	α_4	α_5
45°00’05”	45°00’10”	44°59’58”	45°00’07”	44°59’54”

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

10

Total Marks: 100