CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING <u>March 2015</u>

Note: This examination consists of 7 questions and formulae on 8 pages.

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Q. No	Time: 3 hours	Marks Value	Earned
1.	Often, provincial or other authorities require that measurements, s_{ij}, by an EODMI or total station be done on a <u>calibration baseline</u> that has known pillar coordinates, $\underline{\mathbf{x}} = [x_1, x_2, \ldots, x_n]^T$ with covariance, $\mathbf{C_x}$. Many of the calibration baselines were established several decades ago when EODMI were ± 5 mm ± 5 ppm, compared to ± 1 mm ± 2 ppm commonly encountered today. Rather than using a calibration baseline, some aspects of EODMI behaviour can be investigated by using an *ad hoc* <u>collinear array</u> of points such as a series of tribrachs on tripods. ISO standard 17123-4 requires an array of 7 points [21 one-way distances being observable] with spacing following the Heerbrugg design, as explained by Rüeger in his *<u>Introduction to Electronic Distance Measurement</u>*. The spacing is based on the unit length [$U = \lambda_{mod}/2$] of the EODMI and on the overall length of the array, which is usually at least as long as any intended use of the EODMI. An *ad hoc* array means that the $\underline{\mathbf{x}}$ would not be known *a priori*. In either the baseline or the array, the redundancy allows for a least squares linear parametric estimation. Compare the use of a 7 point collinear array to the use of a 7 pillar calibration baseline under each of the following considerations:		
	a) What quantities are "observed" [values recorded in the field];	1	
	b) What corrections are applied as "pre-processing" [i.e., before the estimation] to the values that are raw output from the EODMI and why;	3	
	c) What quantities are estimated and a typical observation equation [relating observables to estimables] with an explanation of the variables;	4	
	d) Explain the statistical tests that are usually done *a posteriori* [null and alternate hypotheses, statistic, test];	2	
	e) Using a 7 point linear array, a value of -0.93 mm was estimated for the additive constant. Its standard deviation was estimated to be ±0.43 mm. Is the additive constant significant at 95%?	2	
	f) Explain how the results of the estimation and the statistical tests would be used in subsequent use of the EODMI;	1	
	g) Explain the advantages and disadvantages of using a 7 point collinear array rather than a 7 point calibration baseline.	2	

2.	A local plane coordinate system was established at the collar of a shaft at a latitude of 60°59'N. At a depth of 1.0 km, an adit runs approximately in a westerly direction. A flat traverse follows the adit with stations along one side. Gyro-azimuths, using the transit method, have been observed at regular intervals in order to "control" the orientation of the adit. The transit method results in an angle, A_g, describing the direction of the gyro zero with respect to North. The equipment and procedures suggest that $\sigma_{Ag} = \pm 3"$. Explain the corrections, with suggestions of their values [especially signs], which should be applied to an A_g, observed at 4.5 km [westerly] from the shaft in order to convert it to a grid azimuth in the surface coordinate system. If you are not able to calculate a value for a correction, explain what other information would be needed to do so and how it would be obtained.	10	
3.	A campaign of observations, at t_1, can be adjusted to estimate the coordinates of the points involved, based on $\mathbf{l}_1 + \mathbf{v}_1 = \mathbf{A}_1\mathbf{x}_1$ ["**l**" etc. (bold lower case) denote vectors, "**A**" (bold) denotes a matrix]. During a later campaign, at t_2, the observations can be repeated so that $\mathbf{l}_2 + \mathbf{v}_2 = \mathbf{A}_2\mathbf{x}_2$. If there are object points on a sensitive structure, its behaviour can be described geometrically with respect to the reference points, using the displacement field resulting from $\mathbf{d}_x = \mathbf{x}_2 - \mathbf{x}_1$, in which some of the elements of the $\mathbf{d}_x$ vector are for the object points, say $\mathbf{d}_{x\,obj}$. Under certain circumstances, it may be possible to difference the observations, $\mathbf{d}_l = \mathbf{l}_2 - \mathbf{l}_1$, so that the displacement field can be estimated more directly, based on $\mathbf{d}_l + \mathbf{v}_d = \mathbf{A}\mathbf{d}_x$. a) Explain the geometric conditions [especially: configuration, datum] under which the $\mathbf{d}_x$ can be calculated in the coordinate differencing approach, with respect to the $\mathbf{l}_i$, $\mathbf{A}_i$ and $\mathbf{x}_i$, and the advantages and disadvantages of this approach. b) Explain the geometric conditions [especially: configuration, datum] under which the observation differencing approach may be followed and its advantages and disadvantages. c) Explain which approach can accommodate geotechnical data and give an example of an appropriate geotechnical observable, r_j, with its observation equation [relating observable to estimables] and an explanation of how the value recorded in the field, r'_i at t_i, becomes the "observation" value, r_j. d) If the monitoring were to endure over a long period of time, say several decades, explain what concerns would arise in each of the two approaches and how best to deal with those concerns.	4 4 4 3	
4.	A flat hanging traverse is to be measured with uniform sight lengths of 110 m ±2 mm. There are two "fixed" stations, "A" and "B", plus six traverse stations, "P1" to "P6" so that "B" and "P1" to "P5" would be occupied while "A" and "P6" would be sighted. All stations are at practically the same elevation. One approach is to measure the included horizontal angles [values near 180°] with $\sigma_\beta = \pm 5"$. An alternative method is to occupy certain stations and to observe the azimuth to the next station using a gyro attachment so that $\sigma_A = \pm 15"$. a) If only included angles were observed, explain the dominant component of the random positional uncertainty at the end point of the traverse, "P6", and suggest a value and orientation of the uncertainty. b) If azimuths rather than included angles were observed, explain the dominant component of the random positional uncertainty at the end point of the traverse, "P6", and suggest a value and orientation of the uncertainty. c) If the traverse were along a tunnel, close to one wall, explain what would be the dominant systematic influence and whether included angles or azimuths should be observed.	5 3 2	

5.	As part of a special traverse of "n" angles around a city block, a total station is to be set up along one side of the block, at one station with sight distances of 40 m and 250 m with the angle being very close to 180°. The 40 m sight is nearly horizontal but the 250 m sight is at a slope of 25%. These are the extreme values for this situation. a) Accounting for the effects of centering, leveling, pointing, and reading at this occupation, recommend an instrument that would be capable of meeting the requirement that the block angular misclosure is not to exceed $n^{1/2}10''$. "Not to exceed" is to be regarded as being at 99%. The values taken in the calculation of the misclosure would the averages from at least two sets [a set being the average from face left and face right sightings]. b) Explain which of centering, leveling, pointing, or reading is the dominant influence in this occupation and why.	15 5	
6.	On the shelf in the company's survey stores, you have found a total station that has not been used for at least 15 years. The manufacturer's claim, following DIN 18723 [or ISO 17123, now], is an angular "accuracy", horizontally or vertically, of $\pm 1''$ and a distance "accuracy" of $\pm$ 1 mm $\pm$ 1 ppm. Since there is no record of any testing or calibration of this particular instrument, explain the steps that you would recommend following to determine whether this total station is capable of behaving as the manufacturer claimed or better.	15	
7.	Surveys and Mapping Branch [1978], in its _Specifications and Recommendations for Control Surveys and Survey Markers_, specifies that Special Order Levelling should use an optical mechanical instrument [sensitivity of $10''/2$mm, magnification of at least 40x] with a parallel plate micrometer along with double scaled invar staves [usually with 10 mm graduations]. Lengths of sight are not to exceed 50 m and are to be balanced within 5 m. Also, for Special Order, the discrepancy between independent forward and backward levellings is not to exceed [i.e., $1-\alpha = 99\%$] $\pm 3\text{mm}K^{1/2}$ with K in kilometres. First Order levelling is similar but with 32x, 60 m, 10 m, and $\pm 4\text{mm}K^{1/2}$. Several instruments with appropriate staves are available: i. Wild N3 tilting level [42x, $10''$/div (setting accuracy $\pm 0.25''$), σ_{ISO}: ± 0.2 mm]; ii. Kern GK23 tilting level with micrometer [30x, $20''$/div, σ_{ISO}: ± 0.5 mm]; iii. Kern GK2A automatic level with micrometer [33x, σ_{ISO}: ± 0.3 mm]; iv. Leica NA2 automatic level with micrometer [32x, σ_{ISO}: ± 0.3 mm]; and v. Leica DNA03 digital automatic level [24x, setting accuracy $\pm 0.3''$, σ_{ISO}: ± 0.3 mm] with bar coded invar staves. The "σ_{ISO}" is the standard deviation per 1 km of double run levelling according to ISO 17123-2. a) Consider each type of level and, with some numerical substantiation, explain whether it is suitable for Special Order levelling and, if not, whether for First Order. b) Explain how you would ensure that the lengths of sight are balanced in Special Order levelling [50 m $\pm$ 5 m] along a route with many successive setups along a substantial slope.	10 5	
	Total Marks:	100	

Percentiles of the χ^2 distribution:

	0.90	0.95	0.975	0.99	0.995
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84

Percentiles of the t distribution:

	0.90	0.95	0.975	0.99	0.995
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819

Some potentially useful formulae are given below.

$$\sqrt{\sigma_c^2} \approx \pm 0.001h; \sqrt{\sigma_c^2} = \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0005h; \sqrt{\sigma_c^2} \le \pm 0.0001$$

$$\sigma_{\delta_c}^2 = \frac{\sigma_{c_F}^2 + \sigma_{c_T}^2}{s_{FT}^2}$$

$$\sigma_{\beta_C}^2 = \frac{\sigma_{c_F}^2}{s_F^2} + \frac{\sigma_{c_T}^2}{s_T^2} + \left[\frac{1}{s_F^2} + \frac{1}{s_T^2} - \frac{2}{s_F s_T}\cos\beta\right]\sigma_{c_A}^2$$

$$\sigma_l = \pm 0.2div; \;\; \sigma_l = \pm 0.02div; \;\; \sigma_l \le \pm 0.5"$$

$$\sigma_{\beta_l} = \pm \sigma_l \sqrt{\cot^2 z_i + \cot^2 z_j}$$

$$\pm \frac{30"}{M} \le \sigma_p \le \pm \frac{60"}{M}; \qquad \sigma_{ps} \approx \frac{70"}{M}$$

$$b = 2a + c; \quad a = \frac{120}{206264.8}\frac{D}{M}; \quad 2" \le c \le 4"$$

$$\sigma_r \ge \pm 0.3div; \sigma_r = \pm 0.3div; \sigma_r = \pm 2.5div; \sigma_r = \pm 0.6"$$

$$\sigma_z^2 = \sigma_{z_l}^2 + \sigma_{z_p}^2 + \sigma_{z_r}^2$$

$$\sigma_{z_l} = \pm \sigma_l$$

$$\sigma_{z_p} = \pm \frac{\sigma_p}{\sqrt{2}}$$

$$\sigma_{z_r} = \pm \frac{\sigma_r}{\sqrt{2}}$$

$$\sin\beta_1 = \frac{b_1 \sin\alpha_1}{a}; \quad \sin\beta_2 = \frac{b_2 \sin\alpha_2}{a}$$

$$\sigma_\beta^2 = \frac{\tan^2\beta}{b^2}\sigma_b^2 + \frac{\tan^2\beta}{a^2}\sigma_a^2 + \left(\frac{b^2}{a^2\cos^2\beta} - \tan^2\beta\right)\sigma_\alpha^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_n - x_i)^2 \sigma_{\beta_i}^2; \quad \sigma_{y_n}^2 = \sum_{i=1}^{n-1}(x_{i+1} - x_i)^2 \sigma_{\alpha_i}^2$$

$$\sigma_{x_n}^2 = \sigma_s^2 \sum_{i=1}^{n} \left(\frac{x_i - x_{i-1}}{s_i} \right)^2$$

$$\sigma_{x_n}^2 = \sum_{i=1}^{n-1} (y_n - y_i)^2 \sigma_{\beta_i}^2 + \sum_{i=1}^{n-1} \left(\frac{x_{i+1} - x_i}{s_i} \right)^2 \sigma_{s_i}^2$$

$$\sigma_{y_n}^2 = \sum_{i=1}^{n-1} (x_n - x_i)^2 \sigma_{\beta_i}^2 + \sum_{i=1}^{n-1} \left(\frac{y_{i+1} - y_i}{s_i} \right)^2 \sigma_{s_i}^2$$

$$\sigma_{x_n y_n} = \sum_{i=1}^{n-1} (y_n - y_i)(x_n - x_i) \sigma_{\beta_i}^2 + \sum_{i=1}^{n-1} \left(\frac{(x_{i+1} - x_i)(y_{i+1} - y_i)}{s_i^2} \right) \sigma_{s_i}^2$$

$\sigma_s^2 = a^2 + b^2 s^2$

$$d\delta = 8'' \frac{pS}{T^2} \frac{dT}{dx}$$

1 atm = 1013.25 mb = 101.325 kPa = 760 torr = 760 mmHg
0 C = 273.15 K

$$T = \frac{\sum_{i=1}^{n} \left[(h_{i+1} - h_i)(T_i + T_{i+1}) \right]}{2(h_n - h_1)}$$

$$\Delta h_w = \frac{w}{aE} \left(Lh - \frac{h^2}{2} \right)$$

$$n_a = 1 + \frac{0.359474(0.0002945)p}{273.15 + t}$$

$$n_a = 1 + \frac{0.359474(0.0002821)p}{273.15 + t}$$

$$\Delta N_1 = 294.5 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\Delta N_1 = 282.1 - \frac{0.29065p}{1 + 0.00366086t}$$

$$\varepsilon_A = \frac{206264.8}{b} \sqrt{e_1^2 + e_2^2}$$

$$e_i^2 = \left[\frac{e}{2} \right]^2 + [2r]^2 + [0.2mm]^2$$

$$\Delta H = \frac{PH}{aE};\qquad E = 2.1\text{x}10^6\ \text{kgcm}^{-2}$$

$$T = 2\pi\sqrt{\frac{H}{g}};\qquad g = 980\ \text{cms}^{-2}$$

$$e = \frac{30hHdv^2}{P}$$

$$r_0 = r_2 - \frac{P_1(r_1 - r_2)}{P_2 - P_1}$$

$$r = \frac{\pi d^4 E}{64RP}$$

N = N' + ΔN; ΔN = caΔt; E = A - A_g = t ± γ - A_g

$$\alpha_1 = \mathrm{A} - \eta\tan\phi$$

$$z = Z + [\xi\cos\alpha_1 + \eta\sin\alpha_1]$$

$$\alpha_2 = \alpha_1 + [\eta\cos\alpha_1 - \xi\sin\alpha_1]\cot z$$

$$\alpha_3 = \alpha_2 + \frac{h}{M_m}e^2\sin\alpha_2\cos\alpha_2\cos^2\phi_{TO}$$

$$\alpha = \alpha_3 - \frac{e^2s^2\cos^2\phi_m\sin 2\alpha_3}{12N_m^2}$$

$$t = \alpha - \gamma - [T - t]$$

$$t = \alpha - \theta - [T - t]$$

$$\theta = \frac{d\tan\phi(1 - \varepsilon^2\sin^2\phi)^{\frac{1}{2}}}{a}$$

$$\Delta\gamma = \frac{\Delta E\tan\phi}{R}$$

6378206.4 m, 0.0822718948;   6378137.0 m, 0.081819191

$$\varepsilon = \frac{\Delta\ell}{\ell} = \frac{\Delta s}{s}$$

$$d_x = r_{x_1} - r_{x_2};\quad d_y = r_{y_1} - r_{y_2}$$

$$\theta_x = \frac{d_x}{s}; \quad \theta_y = \frac{d_y}{s}$$

$$c = [N_0 - N_a]s$$

$$c_{cal} = \frac{s_{std} - s_{obs}}{s_{std}} s_i; \; c_{align} = -\frac{d^2}{2s}; \; c_{temp} = \alpha(t - t_0)s; \; c_{tens} = \frac{P - P_0}{aE} s$$

$$c_{sag} = -\frac{s^3}{24}\left(\frac{mg\cos\theta}{P}\right)^2\left(1 \pm \frac{mgs\sin\theta}{P}\right); \quad c_{sea} = \frac{H}{R+H} s$$

$$\frac{s^2}{\sigma^2} \le \frac{1}{\nu}\chi^2_{\nu,1-\alpha}?; \quad \frac{1}{F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}} \le \frac{s_1^2}{s_2^2} \le F_{\nu_1,\nu_2,1-\frac{\alpha}{2}}?; \quad \frac{a_\mu}{s_{a_\mu}} \le t_{\nu,1-\frac{\alpha}{2}}?$$

$$\left|\frac{\hat{r}_i}{\sigma_{\hat{r}_i}}\right| \le n(0,1), 1-\frac{\alpha}{2}; \qquad \left|\frac{\hat{r}_i}{\hat{\sigma}_{\hat{r}_i}}\right| \le \tau, \nu, 1-\frac{\alpha}{2}, \quad \tau_\nu = \frac{\sqrt{\nu}}{\sqrt{\nu - 1 + t^2_{\nu-1}}} t_{\nu-1}$$

$$C_x = \sigma_0^2\left[C_{x_S}^{-1} + \left(A^T P A\right)_U\right]^{-1}$$

$$\Delta_{f/b} \le \pm 3mm\sqrt{K}; \; \Delta_{f/b} \le \pm 4mm\sqrt{K}; \; \Delta_{f/b} \le \pm 8mm\sqrt{K}; \; \Delta_{f/b} \le \pm 24mm\sqrt{K}$$

$$\sigma_{r_l} = \pm d\sigma_l; \qquad \sigma_{r_{pr}} = \pm\frac{45''}{M}d, \quad d > 20m; \quad \sigma_{r_{pr}} = \pm\frac{30''}{M}d, \quad d \le 20m$$

$d_{y1} = r_{1,1} - r_{2,1}$; $d_{y2} = r_{1,2} - r_{2,2}$; $\Delta y = d_{y2} - d_{y1}$

$$T = \frac{\Delta y}{\Delta H}$$

$$s_{ij} + z_0 = x_j - x_i; ks_{ij} + z_0 = x_j - x_i; s = s' + s'\Delta N$$

$$n_{obs} = \frac{n_{pts}\left(n_{pts} - 1\right)}{2}$$

$c+r = 0.0675\ K^2$

$$A = iU; \quad B_0 = \frac{1}{15}[C_0 - 6A - U]; \quad D = \frac{U}{36}$$

$1to2 : A + 1B + 3D$

$2to3 : A + 3B + 7D$

$3to4 : A + 5B + 11D$

$4to5 : A + 4B + 9D$

$5to6 : A + 2B + 5D$

$6to7 : A + D$

$$d_4 = 2R\arcsin\sqrt{\frac{R^2\sin^2(d_1\frac{k}{2R}) - k^2\frac{(H_2-H_1)^2}{4}}{k^2(R+H_1)(R+H_2)}}$$

$$d_4 = R\arctan\left[\frac{d_2\sin(z_1+\varepsilon_1+\delta)}{R+H_1+d_2\cos(z_1+\varepsilon_1+\delta)}\right]$$

$$\hat{y} = a + bx; \qquad x = \frac{-a}{b} + \frac{\hat{y}}{b}; \quad x = z_0 + k\hat{y}$$

$$s_a = s_0\sqrt{\frac{\sum x^2}{n\sum x^2 - \left(\sum x\right)^2}} \quad \textit{and} \quad s_b = \frac{s_0}{\sqrt{\frac{\sum x^2 - \left(\sum x\right)^2}{n}}} \quad \textit{with}$$

$$\hat{\sigma}_0^2 = s_0^2 = \frac{\sum\left(a+bx-y\right)^2}{n-2}$$

$$\hat{\sigma}_{z_0}^2 = \hat{\sigma}_0^2\frac{6}{\left(N-1\right)\left(N-2\right)} \quad \textit{with} \quad \hat{\sigma}_0^2 = \frac{\sum v^2}{n-u}$$